GEODETIC ASPECTS OF LAND BOUNDARIES AND THE PLSS DATUM IN A CADASTRAL COMPUTATION SYSTEM

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ABSTRACT

Many boundaries and most elements of the Public Land Survey System are defined in a geodetic sense, for example lines of constant true bearing, latitudinal arcs, meridians, long straight lines, parallel and other equidistant lines. This paper will discuss the implications of these definitions, giving several examples, and discuss the challenge of how to properly deal with the unique characteristics of such boundaries. This paper will discuss how this challenge has led to the development of a PC based computational system for use in the Bureau of Land Management's Cadastral Survey organization.

BACKGROUND

Computational methods

In the current world of surveying there are two widely varying computational methods in common use. The first method is simple plane survey computations performed on a local orthogonal coordinate system. Another method in use for control survey applications uses geodetic computations on a variety of geodetic systems using spherical or ellipsoidal coordinate systems (latitudes and longitudes). One common variation of geodetic computations is the use of any number of coordinate projections or grids.

It is generally understood by surveyors that the use of simple plane methods while convenient, is not necessarily suitable for large scale surveys. Why this is so is not usually so well understood. Plane survey computations have become associated with almost all boundary and construction surveying while geodetic methods are most often associated with control surveying, mapping and route surveying. However there are many large scale boundary surveys where this distinction between computational methods cannot be maintained. In many large scale surveys it becomes necessary to deal with some geodetic aspects of the survey. In addition a historical look at the methods used to carry out such surveys reveals some interesting information that affects how we must deal with retracing them today.

A good example of a survey system with significant geodetic components is one used throughout the western United States, the Public Land Survey System (PLSS). The PLSS was devised in a period of time where survey instrumentation was limited, yet vast areas of land needed survey. The system was modeled after a practical survey system that had been used successfully in New England. The system quickly evolved to a framework of rectangular townships, often divided into smaller parcels. As the PLSS was prototyped and implemented beginning in Ohio, it evolved over it's first 50 years. It became quickly apparent that the term `rectangular' is a generality that cannot effectively be maintained over a large survey extent.

Cooperative work between Cadastral Survey of the Bureau of Land Management (BLM), and the University of Maine has resulted in development of an integrated Cadastral Measurement Management (CMM) software system specific to retracement cadastral surveys in the PLSS. One of it's design goals was to provide a set of computational tools that deal properly with these geodetic like characteristics of large scale boundary surveys. The development of CMM was necessary because no available software package existed which effectively worked with this type of information. The following discussion describes some of these special requirements.

THE PLSS DATUM

The reference system by which the measurements of a system are reported can be described as a datum. Such a reference system can be quit involved as in the case of a control survey datum like NAD83, or it can be less comprehensive. In boundary surveying and retracement it is very important to gain an understanding of the basis of the measurements reported in the survey, and in our case we are concerned with what this reference is for the PLSS. This reference for measurement and reporting will be referred to as the *PLSS Datum*. The data being reported on a current Bureau of Land Management (BLM) or older Government Land Office (GLO) Cadastral Survey plats are bearings and distances. But what are these bearings and distances in reference to? The datum that surveys of the PLSS have been reported in for over 200 years is discussed in various sections of the Bureau of Land Management's current guide, *The Manual of Surveying Instructions, 1973*, as summarized below.

Reference to Manual Sections

- *1-3. "Details of the plan and its methods go beyond the scope of textbooks on surveying. The application to large-scale area requires an understanding of the stellar and solar methods for making observations to determine the true meridian, the treatment of the convergency of the meridians, the running of the true parallels of latitude, and the conversion in the direction of lines so that at any point the angular value will be referred to the true north at that place."*

  2-1. "The law prescribes the chain as the unit of linear measure for the survey of the public lands. All returns of measurements in the rectangular system are made in the true horizontal distance in miles, chains and links...."

  2-17. "The direction of each line of the public land surveys is determined with reference to the true meridian as defined by the axis of the earth's rotation. Bearings are stated in terms of angular measure referred to the true north or south."

  2-19. Describes 3 Basic methods practices used to determine true bearings, 1) Direct Sun, star or polaris observations; 2) Solar attachment; 3) Angles from the horizontal control network. Use of the gyro-theodolite is also mentioned.

  2-74. This entire section is very important in describing geodetic implications of the PLSS datum. It describes in detail methods for carrying forward true bearing, and defines the term mean bearing and many other critical concepts. ".... By basic law and the Manual requirements, the directions of all lines are stated in terms of angular measure referred to the true north (or south) at the point of record."

  2-76. Defines the solar method for laying out the true parallel of latitude.

  2-79. Contains formulas for computing Convergency of two meridians, corrections to closure computations as a factor of area, and computing Rb, effective radius of a parallel.

  2-80. Describes use of the Standard Field Tables to determine convergency and other geodetic effects.

  2-81,82. Describe the use of M and P factors for converting measurements to differences in latitude and longitude. A simple version of geodetic computation.

3-87. Subdivision of Sections: use of the term "straight lines".

5-25. Double proportion - 'Cardinal equivalents to be used, and cardinal offsets made to determine final point.

5-31. Single Proportion: correction for latitudinal curve.

5-36. Other adjustments to allow for curvature.

These references as well as those in other BLM and GLO Manuals make it clear that we can define the PLSS datum with the following observations.

Distances: The frame of reference for distances is defined as ***horizontal measure in chains based on the U.S. Survey Foot at actual ground elevation***. This is true because measurements were made along the ground with reduction to horizontal only. With a chain length of from 2 poles to 5 chains this effectively follows the terrain.

Bearings: The frame of reference for direction in the PLSS as something called ***Mean True Bearings*** referenced to the true meridian *"... at the point of record"*. There is a small ambiguity in this definition, in that there is a slight difference between a geodetic azimuth and an astronomic azimuth that has generally been neglected considering the overall accuracy and instrumentation used.

Figure 1: Lines on exagerated converging meridians.

Those familiar with basic geodesy will recognize that this is a basis of bearing that is not orthogonal. In one way of thinking it is a reference that changes as you go east and west because these reference meridians are not parallel but converge towards the pole.

Because this is a non-orthogonal reference the direction of a straight line on the ground can be described with a forward bearing based on the meridian at the beginning point, or by a different back bearing based on the meridian at the end point. The difference between these is the angle of convergency of the meridians through those end points.

To visualize this refer to *Figure 1*, which illustrates a simple model of the bearing datum with meridians converging as you go north. A straight line from Point "A" to the right to point "B" on the left (line 1), crosses each meridian at a differing angle. The angle with a meridian is the true bearing at that point. More on this later.

To describe how far north or west the line goes in a geodetic sense, the average or 'mean' of these two values is used. This *mean bearing is essentially identical to the bearing of the straight traverse line with reference to it's midpoint. Thus the point of record* for determining the bearing of a traverse line can be said to be the meridian at the midpoint of the line.

IMPLICATIONS OF THE PLSS DATUM

Straight Lines

A straight line for our purposes is what most people would think of as a straight line. It is the same as a line of sight, or a line laid out by double centering. In a purist geodetic sense a long double-centered line is not exactly straight due to small variations in gravity as well as the effects of travelling along an near elliptical surface. Over very long lines we would end up with a slightly wavering line called a geodesic. For practical considerations in the PLSS however a line of sight is a straight line. The first unusual byproduct of the PLSS datum is:

- **I.** Straight lines on the ground are lines of constantly changing bearing.

This is true because such a line will pass each converging true meridian at a different angle. (Unless it is a due north and south line which goes along the meridian.)

An example of a boundary that might be straight is one described as a straight line running from one physical monument or point to another. The South boundary of California (with Mexico) and the diagonal eastern boundary between California and Nevada are examples. Such a line if reported in the PLSS Datum would have considerably different forward bearing and back bearings as well as different bearings at each point along it. This is illustrated by line 1 in *Figure 1*, between points "A" and "B". This line has a forward bearing at "A" of WEST, it departs at right angles to this meridian. As one moves along the line to the west it crosses each meridian an a stronger angle going farther and farther to the southwest. The back azimuth is the angle with the meridian at point "B".

Lines of constant bearing

The actual methods and instruments that were used to lay out many survey lines determined bearing at frequent intervals with reference to the converging meridians, and were thus run at a constant bearing.

Many boundary lines in the PLSS are in fact lines of constant true bearing. Another term used in navigation is a *rhumb* line. It is important to realize that these lines are *NOT* straight lines as they cross every meridian at the same angle and thus appear curved as viewed on the ground. Therefore:

- **II**. Lines of constant bearing in the PLSS datum will be `curved' on the ground.

The magnetic compass, solar compass and solar transit determined true meridian at each setup. When this type of instrumentation is matched with traditional chaining of the lines they laid out lines of constant bearing which are therefore curved.

The *`Manual'* discussion of latitudinal arcs illustrate one example of a rhumb line. A parallel of latitude is a line that is due East and West in the PLSS Datum. Such a line must cross each meridian at a 90 degree angle in order to have a mean bearing of East or West. An example of such a line is illustrated by line 3 in *Figure 1*. This line crosses each meridian at right angles, and is also a parallel of latitude. A line of constant bearing between point "A" and "B" is shown as line 2. It also crosses each meridian at the same angle. It is nearly, but not exactly, a circular curve. All lines of constant bearing in the PLSS datum except meridians will appear curved on the ground.

Chords to the arc

Since survey instruments more easily deal with straight lines and traverse lines are straight lines, a useful concept to keep in mind is that:

- **III.** The mean bearing of any chord or sub-chord connecting any two points along a PLSS rhumb line will be the same as the bearing of the rhumb line itself.

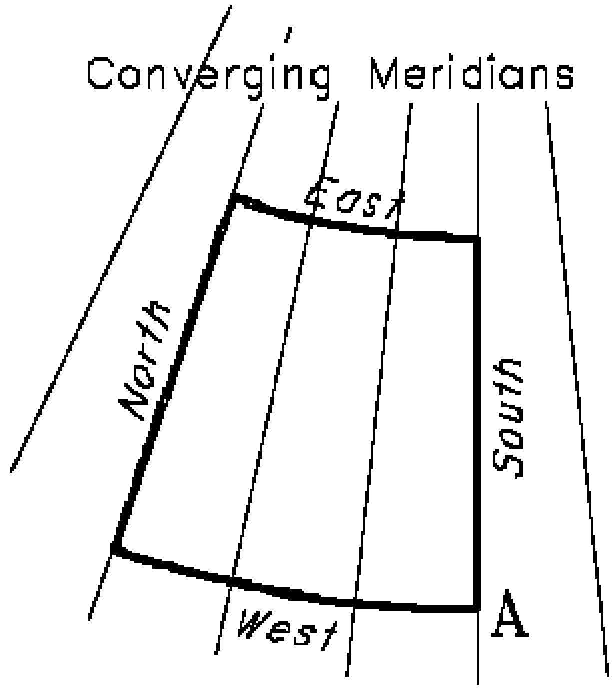
This is illustrated by examining the straight line `chords' *c1, c2, c3* and *c4* shown in *Figure 1*. If these were traverse lines corrected to mean bearings, their endpoints fall along the curved line being run. Thus it is possible to lay out points on a rhumb line by correcting each traverse line to it's mean bearing in computations. It can be seen that the midpoint of each of these lines is at right angles to that meridian and therefore have a mean bearing of East and West.

Most PLSS boundaries are RHUMB lines or lines of constant bearing. This includes standard parallels, township exteriors, section lines, many grant and reservation lines. Other examples include many state lines, the U.S. - Canada border, etc.

Some boundaries are straight lines. This includes specifically described portions of grant or reservation boundaries, some portions of state lines. Subdivision of Section, (see Section 3-87 of the *Manual*).

Closure

Another phenomenon arises when we attempt to compute traverses in this datum. To illustrate this see *Figure 2*. The figure shown overlaying the exaggerated converging meridians is what could be called a



"cardinal square". It is a figure which has its sides bearing West, North, East and South by true meridian bearings. In this case let us say that the South, West and East lines are all 5280.00 feet in length. It is apparent from the illustration that the north line must be shorter due to the convergency of the meridians.

How much the meridians converge is a function of the particular latitude, it being larger the farther north in latitude one goes (in the northern hemisphere).

At a latitude of 40ø at about the middle of the continental U. S., the north line of such a `square' would be 1.12 feet shorter. At a latitude of 70ø in Alaska the line would be 3.65 feet shorter.

Figure 2. A cardinal "square" (above right)

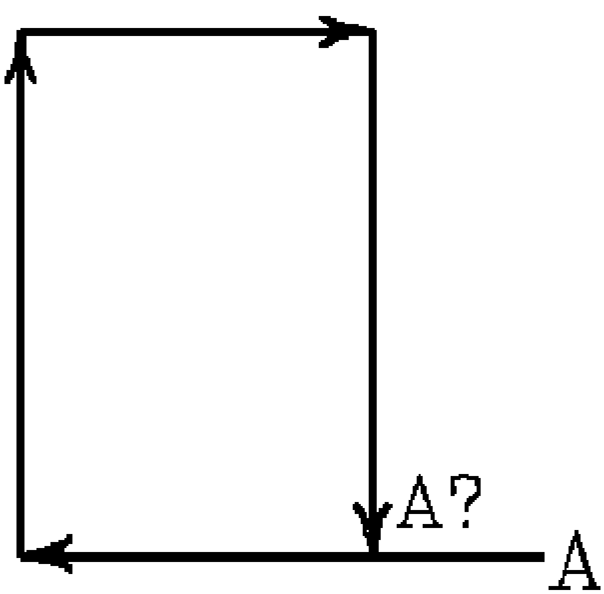
If we were to take these lines and attempt to compute a closure, we would see something like the diagram shown in *Figure 3*. As you can see one does not appear mathematically to return to the place of beginning even though this is a perfect geometric figure. This apparent misclosure is in easting only.

Figure 3 - How the data would close... (below left)

This leads to another observation:

- **IV.** A theoretically perfect survey will appear to misclose in the PLSS datum.

This effect will be called *`apparent misclosure due to convergency of the meridians'*. It is a function of latitude and the area of the loop being closed.



Due to this apparent misclosure a computation of coordinates for a point through traverses will produce a different coordinate value for each path taken . The values will depend on the area between the path to the point and a true line directly to the point. The only correct answer is one obtained by running the straight line to the point. Coordinates systems based on mean bearings will always have difficulty because the math and trig functions we are tempted to use is based on an orthogonal coordinate system, and we are dealing with a different shaped system.

Elevation

The distances in most surveys are reduced to horizontal, but are often not to any other surface. This is true in the PLSS datum as well. Because of this the elevation of the actual lines sometimes becomes relevant in a computation. For example, if the north boundary of a township varies 3000 feet in elevation along it's length, care must be taken in the method used to proportion lost positions for corners along it. This is not a problem when proportioning against a retraced line run over the same area, but could be a problem if, for example, the data had been reduced to a grid, or to `sea-level' latitudes and longitudes before being proportioned. In essence, elevation can be a weighting factor if reduced measurement data is used for the proportion.

SPECIAL BOUNDARY RESTORATION ADJUSTMENTS

In the PLSS a number of specialized corner restoration processes have evolved. Among these are, single and double proportion, the irregular boundary adjustments, the non-riparian meander line adjustment and the grant boundary adjustment. Descriptions of these methods are outlined in Chapter 5 of BLM's *Manual of Surveying Instructions, 1973*. All of these methods have important elements that involve the geodetic nature of the PLSS. For example, a correction for latitudinal arc is mandated on single proportions and on East-West irregular boundary adjustments. Double proportion also requires proper consideration for true north in order to be properly performed. Few, if any software packages available include any of these restoration methods at all.

CADASTRAL MEASUREMENT MANAGEMENT

The unusual nature of the PLSS datum creates many challenges to a computational system. As mentioned above, Cadastral Survey, Bureau of Land Management (BLM) in cooperation with the University of Maine has developed an integrated computational system that deals properly with the geodetic affects discussed. The software system is called *Cadastral Measurement Management* or **CMM** for short. It contains a comprehensive set of tools to perform functions in the PLSS datum. For example:

CSTUF CMM contains a geodetic computation program called CSTUF which provides many computations using plane, geodetic forward or geodetic mean bearings. The computations within the system are mostly geodetic but results are stored as state plane coordinates in CMM. An example of the necessity for such a package is illustrated by line 4 shown in *Figure 1*. This illustrates a potential computation of an intersection between a boundary defined by a straight line, and one defined by a line of constant bearing. Use of a traditional computational system would result in the intersection of two straight lines, which can be considerable in error to the north of the correct intersection. CSTUF provides many other features including all forms of intersection computations, mean bearing traversing and inversing. In addition it contains a macro capability that can store and replay computations.

ADJUST The specialized boundary adjustments required for PLSS retracement work are handled in CMM by several programs. The program ADJUST provides manual solutions to single and double proportioning, but in addition is the primary tool for computation of 1, 2 and 3 point control restoration methods, the irregular boundary adjustment and the grant boundary adjustment. Like CSTUF, it also includes macro capability to allow storing a problem for recomputation.

PROPORT The CMM program proport is a more automated means to compute single and double proportions. If a project requires a number of such computations, this can become quit time consuming to do with ADJUST as the record must be entered again for each computation. PROPORT makes use of a file which must be created by the system user that reflects the plat being proportioned. Once constructed and verified, this file is used by several CMM programs to allow quick computation of proportions. Even for a single corner, a proportion may be computed a number of times, first to determined search areas, then to evaluate evidence as it relates to the record, and as other controlling corners are recovered the proportions often need to be recomputed based on current status of found controlling corners.

AUTOPROP An even more automated proportioning utility is also provided in CMM. As with PROPORT this system requires a plat record data file to function. It is then capable of proportioning all normal lost section, 1/4 section, 1/16th section corners and subdividing all normal sections very quickly. The primary reason for this program was to provide corner search coordinates that can be quickly updated as evidence is found and evaluated for controlling PLSS monuments. Another CMM utility is then used to flag traverse points that represent controlling monuments, then AUTOPROP will quickly compute the rest of the points.

CMOVE CMM also contains a basic utility to compute what are called corner moves or `ties'. After the final data is derived and all missing corners have been determined or reestablished then CMOVE provides the ability to compute angle, bearing and distance information for setting monuments. Also true line offsets can be computed.

SUMMARY

The measurement systems used in many large scale boundary surveys and in particular the Public Land Survey System lead to a number of unusual considerations which must be dealt with in retracing such surveys. The lines of the PLSS are described by true mean bearings and ground elevation distances in what is defined as the PLSS Datum. This datum has a number of geodetic characteristics which make it difficult to properly work within with traditional computational methods. These unusual factors lead to the conclusion that a specialized computational system is required. Such a system has been developed by Cadastral Survey, Bureau of Land Management in cooperation with the Survey Engineering Department, University of Maine. This system, called Cadastral Measurement Management, or CMM, also provides many unique computational tools that are necessary for retracement work in the PLSS system and that are not available in any known commercial software package.

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