

BASICS OF LEVELLING

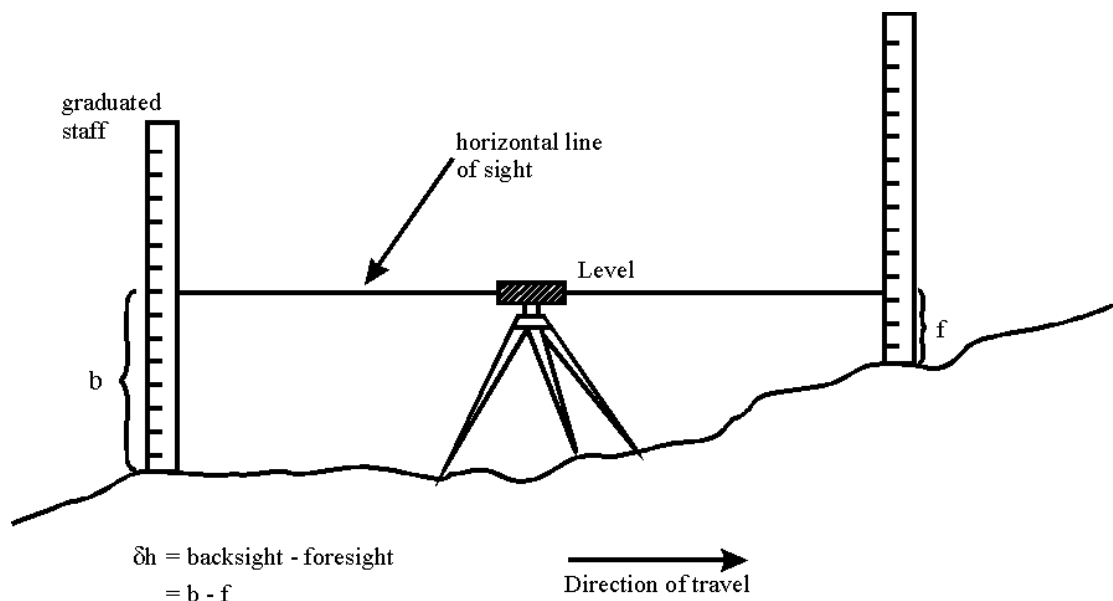
USES OF LEVELLING

In the context of tidal measurements, levelling is used for the following purposes:

- **Referencing of Tide Gauges:** To determine and check the vertical stability of the tide gauge bench mark (TGBM) with respect to reference points (benchmarks) in its immediate vicinity. In order to isolate any local movements, there should be at least three such benchmarks, and the levelling should be repeated on an annual or semi-annual basis.
- **Connection to GPS Reference Points:** To determine its regional stability and to separate sea level rise from vertical crustal motion, the TGBM should be connected via GPS to reference stations fixed in a global co-ordinate system. Generally speaking, the GPS antenna cannot be directly placed on the TGBM and a GPS reference point must be established a short distance away. This must be connected to the TGBM by levelling.
- **Connection to National Levelling Network:** Mean sea level is used to define vertical datums for national surveying and mapping - hence the TGBM must be connected to the national levelling network. Connection to the network will also allow all tide gauges to be connected to each other, providing information on spatial variations in mean sea level.

PRINCIPLE OF DIFFERENTIAL LEVELLING

Differential levelling provides a means of accurately measuring height differences between points some tens of metres apart. A level is set up on a tripod and levelled so that the line of sight is horizontal:



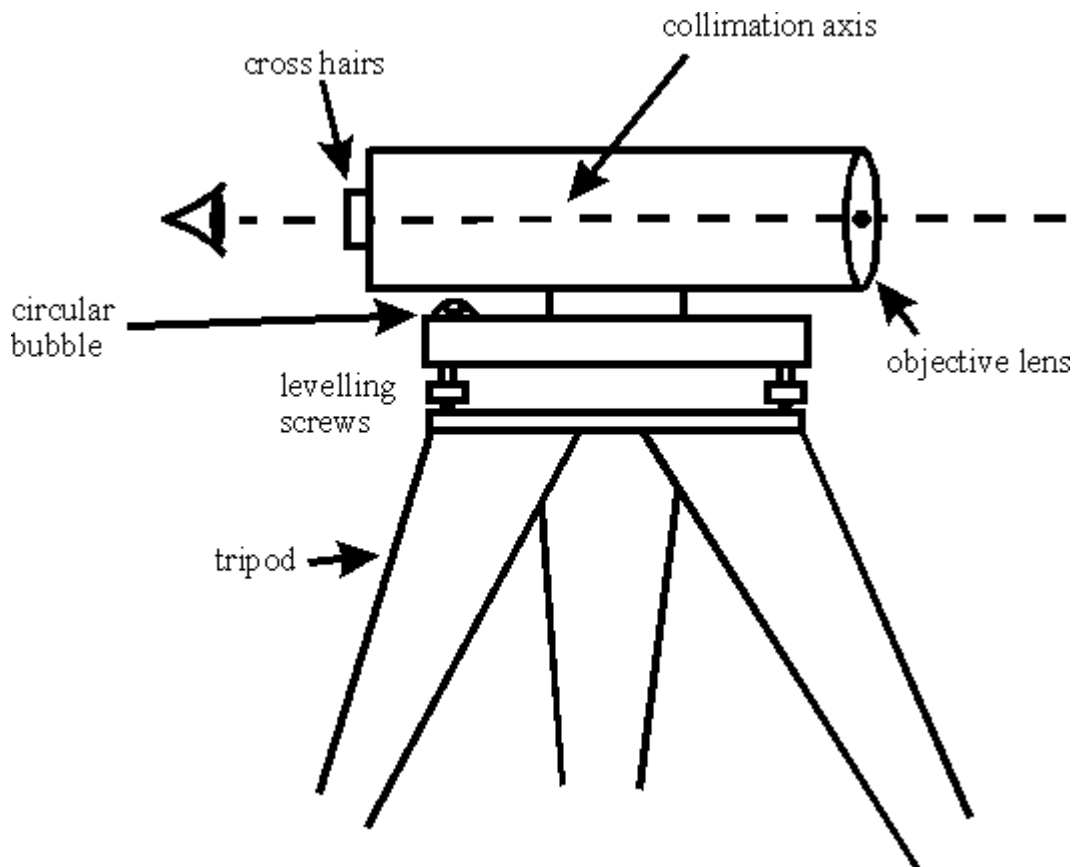
A graduated staff is held vertically over the first point and a reading made of the intersection of the cross-hair with the image of the staff (backsight - b). The same (or an identical) staff is then held vertically over the second point and a further reading made (foresight - f). The difference between the two readings is the difference in height between the two points:

$$\delta h = b - f$$

If b is greater than f then δh is positive (i.e. there is a rise in elevation in moving from the first to the second point).

This process can be repeated - the level can be moved to beyond the second point and the height difference between the second and a third point measured by the same process. Further repetitions will allow the height difference between widely separated points to be determined by accumulating the height differences between (temporary) intermediate points. The distance from level to staff is dictated by the steepness of the terrain and the clarity of the image viewed by the observer. Usually the maximum sight length is restricted to 50-60m.

The sketch below shows a schematic illustration of a basic level:



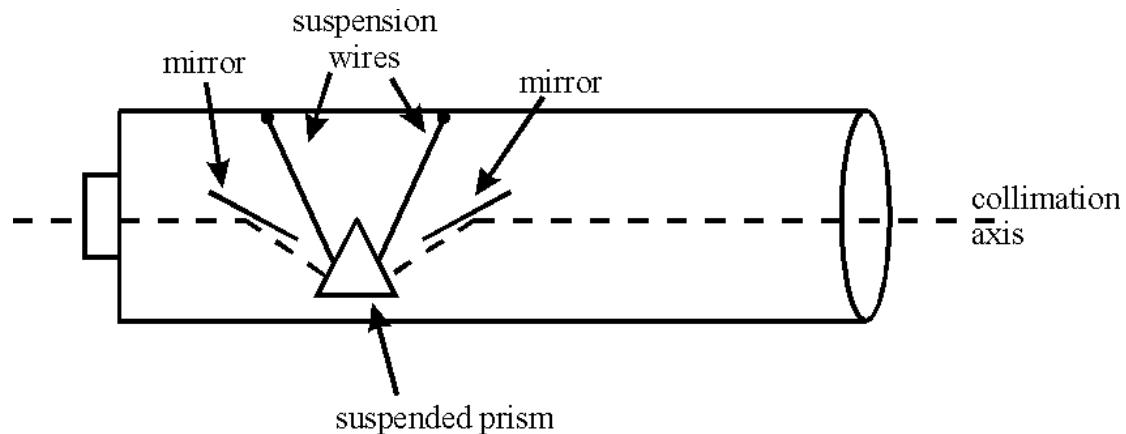
The level is mounted on a tripod, and has three levelling screws that (in conjunction with a circular bubble) allow the level to be levelled. These screws have a limited range and the tripod head must be set approximately level beforehand by adjusting the tripod legs.

The upper part of the level consists of a telescope tube with an objective lens and an eyepiece with a cross-hair. The line of sight (collimation axis) is defined by the line joining the centre of the cross-hairs with the focal point of the objective lens. The telescope is mounted on an axis that allows it to be rotated in the horizontal plane.

The circular bubble is not very sensitive and is not the sole means of levelling the level. Older levels will have tubular bubbles attached to the side of the telescope, and the footscrews are used to level this bubble, which then provides a horizontal line of sight in the direction of the collimation axis.

Automatic Compensator:

Modern levels will all use some form of automatic compensator, which allows the user to level the instrument with the circular bubble only. Any small departures are compensated by the compensator. The figure below shows a schematic illustration of one type of compensator:



In this device the image of the object is deflected by a fixed mirror to pass through a prism, after which it is deflected by another mirror to the eyepiece. The prism is suspended by wires and its orientation changes as the telescope tube is tilted. The geometry of the device is designed so that any tilt of the telescope tube is compensated by a tilt of the prism and the collimation axis remains horizontal. The compensator has a limited range (a few minutes of arc) and the level must be levelled reasonably well using the circular bubble before the compensator will work correctly.

Types of Level:

Broadly speaking, there are three classes of level:

- *Builder's/Engineer's Level:* As implied by the name, these are used by builders and engineers. Their design is basically as described earlier, and they use graduated staffs in which the smallest graduation is 1cm. Millimetres must be estimated, and the accuracy of a single reading will be about 2-3mm.

- *Digital Level:* This type of level uses a special bar-coded staff. The image of the staff passes through the objective lens and then via a beam splitter to a photodetector array, where it is digitised. The microprocessor compares this image to a copy of the bar code and calculates the staff reading, which is displayed and/or stored. The sensitivity of the device is such that single reading accuracies of 0.2mm to 0.3mm can be achieved, and sight lengths can be extended up to 100m.
- *Precise Level:* This is a modification of the conventional level in which a parallel plate micrometer is placed in front of the objective lens. This allows the image of the staff graduation to be moved up or down by very small measurable amounts. For sight lengths of under 50m, single reading accuracies of 0.02mm to 0.03mm can be achieved.

As precision improves, so prices increase. It is tempting to use a builder's level for reasons of economy, and many tidal institutions have done so. However, if measured small changes in mean sea level are to be meaningful, the stability of the TGBM must be unquestioned, and accuracies of 1mm or better are desirable for the levelling connection. Precise levels have been used and will continue to be used, but if a new level is to be acquired, the best option would be a digital level.

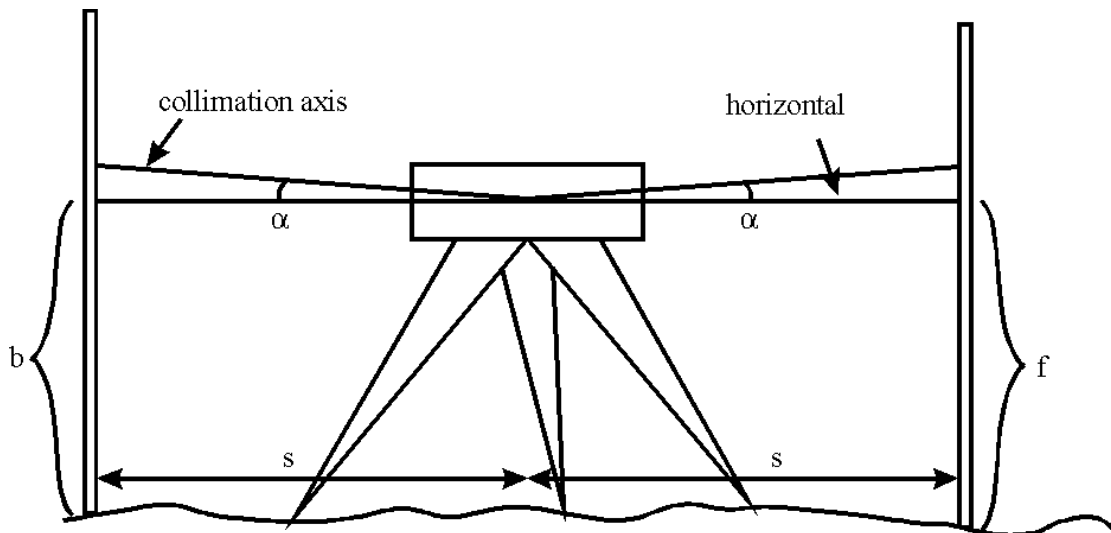
ERRORS IN LEVELLING

There are a large number of potential sources of error in levelling. Many of these are only significant for precise levelling over long distances. For the short segments of levelling that will occur in connecting a TGBM to nearby benchmarks there are only three worth mentioning:

- Collimation Error
- Error due to Earth Curvature
- Error due to Refraction

Collimation Error:

Collimation error occurs when the collimation axis is not truly horizontal when the instrument is level. The effect is illustrated in the sketch below, where the collimation axis is tilted with respect to the horizontal by an angle α :



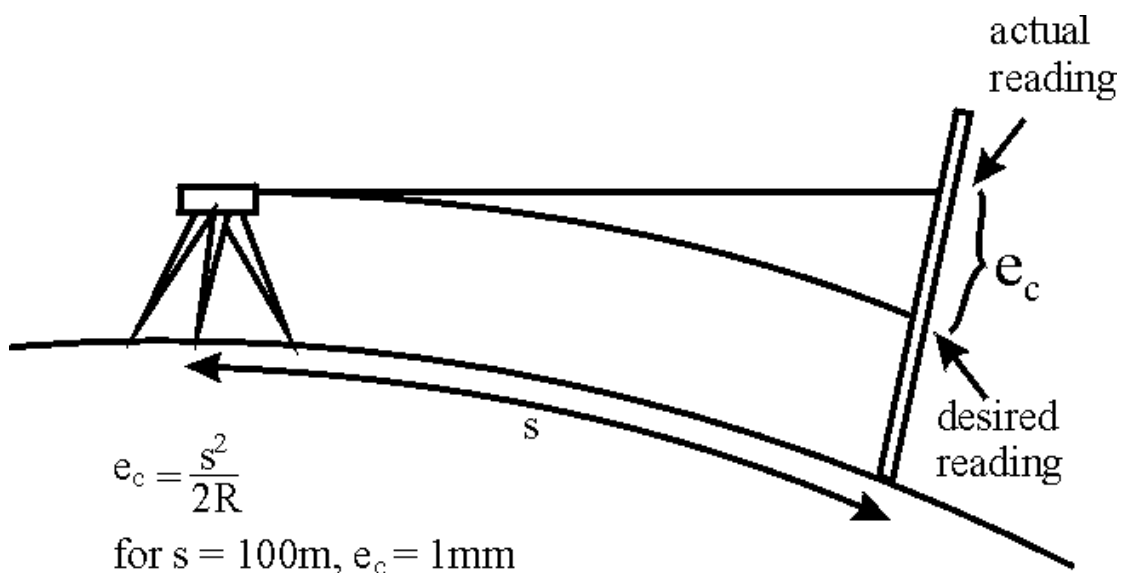
In this particular example, the effect is to read too high on the staff. For a typical collimation error of 20", over a sight length of 50m the effect is 5mm. If the sight lengths for backsight and foresight are *equal*, the linear effect is the same for both readings. When the height difference is calculated, this effect cancels:

$$\delta h = (b + s.\alpha) - (f + s.\alpha) = b - f$$

That is, the effect of the collimation error is eliminated if sight lengths are kept equal.

Earth Curvature:

Due to the curvature of the Earth, the line of sight at the instrument will deviate from a horizontal line as one moves away from the level:



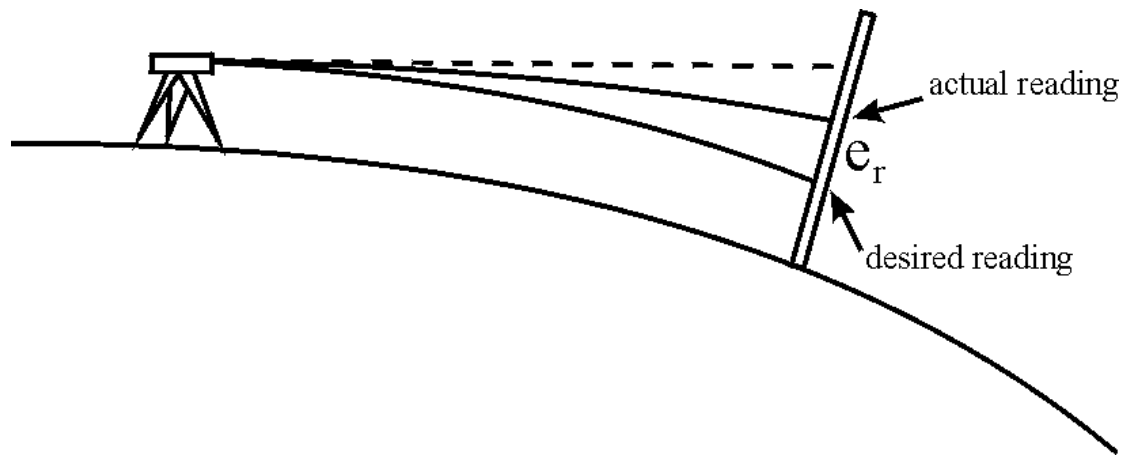
Ideally one would like the line of sight to be a curved line which is everywhere perpendicular to the direction of gravity. The error in staff reading due to Earth curvature is given by:

$$e_c = \frac{s^2}{2R}$$

where s is the sight length and R is the radius of curvature of the Earth. For a sight length of 100m the effect is only 1mm. As with collimation error, the effect is eliminated by using equal sight lengths for fore- and backsights.

Refraction:

The variable density of the Earth's atmosphere causes a bending of the ray from the staff to the level. The effect is illustrated in the sketch below:



The light ray is bent in a path which has a curvature less than that of the Earth's surface, and the combined effect is smaller than that due to Earth curvature alone:

$$e_r = \frac{1-k}{2R} \cdot s^2$$

Here, k is the coefficient of refraction and represents the ratio of the radius of curvature of the Earth to the radius of curvature of the light path. An average value of k is 0.13, from which:

$$e_r = 0.068 \cdot 10^{-3} \cdot s^2$$

where s is in metres and e_r in millimetres. For example, for $s = 100\text{m}$, $e_r = 0.7\text{mm}$.

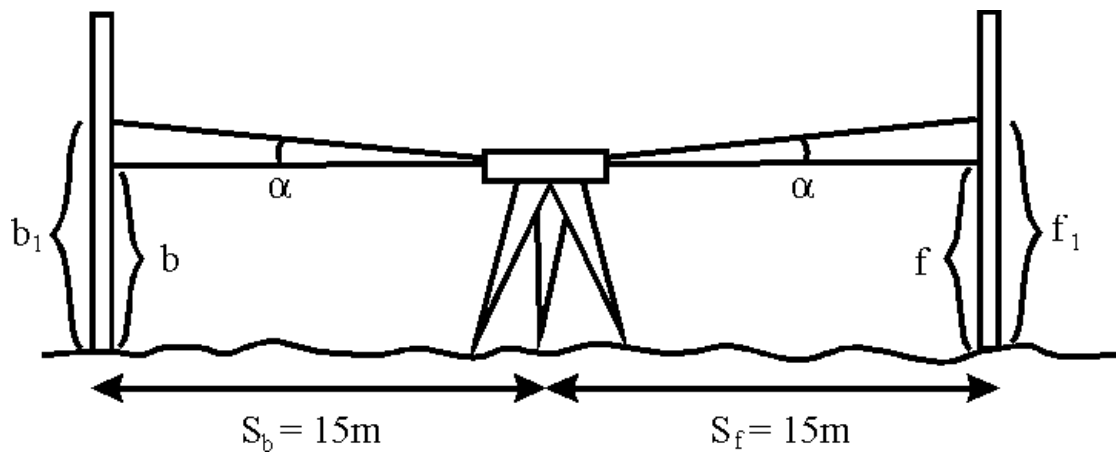
The effect of refraction is almost totally eliminated by using equal fore- and backsights (because atmospheric conditions along the fore- and backsights will not be completely identical, there will be a small residual error).

DETERMINATION OF COLLIMATION ERROR

Collimation error is much more significant than the other errors. It should be kept as small as possible so that one need not be too precise in ensuring that fore- and

backsights are of equal length (these are usually paced out). It is possible to determine the collimation error and reduce its size using the so-called *Two-peg test*. There are three steps involved in this procedure:

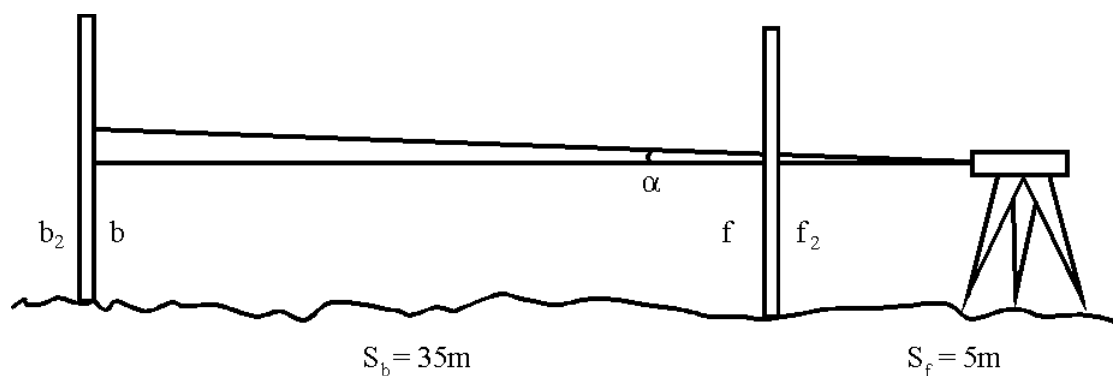
1. Set out and mark on the ground (with wooden pegs driven into the earth, or roofing nails in tar) two points some 30m apart. Set up the level exactly mid-way (within 0.5m) between them:



Take measurements of backsight and foresight for this first setup. The height difference δh_1 will be free of the effects of collimation error:

$$\begin{aligned} \delta h_1 &= b_1 - f_1 = (b + s_b \cdot \alpha) - (f + s_f \cdot \alpha) \\ &= b - f + \alpha \cdot (s_b - s_f) \\ &= b - f \quad (\text{because } s_b = s_f) \end{aligned}$$

2. Next, move the level to a position just beyond the fore staff position (about 5m):



Then repeat the readings. In this case, $s_b = 35\text{m}$ and $s_f = 5\text{m}$. Then:

$$\begin{aligned} \delta h_2 &= b_2 - f_2 = (b + s_b \cdot \alpha) - (f + s_f \cdot \alpha) \\ &= b - f + \alpha \cdot (s_b - s_f) \\ &\neq b - f \quad (\text{because } s_b \neq s_f) \end{aligned}$$

Obviously, this height difference is burdened with the effect of a collimation error over 30m.

3. The difference $\delta h_2 - \delta h_1$ can be used to calculate what the true backsight reading would be for the second setup, if collimation error were not present:

$$b = b_2 - \frac{s_b - s_f}{s_b} \cdot (\delta h_2 - \delta h_1) = b_2 - \frac{30}{35} \cdot (\delta h_2 - \delta h_1)$$

In the case of older levels with tubular bubbles the adjustment consists of tilting the level using the levelling screws until the desired staff reading appears on the cross-hair. Then the adjusting screws on the tubular level are adjusted until the bubble is level. For modern levels with automatic compensators the adjustment involves moving the cross-hairs vertically using their adjusting screws until the desired reading is obtained.

The entire process should be repeated as a check. It is practically impossible to adjust the instrument so that no collimation error exists - the purpose of the adjustment is to reduce the size of this error. If the discrepancy $\delta h_2 - \delta h_1$ can be reduced to around 2mm this is perfectly adequate, provided sight lengths are thereafter kept reasonably similar.

(These notes are based on lectures by Professor Charles Merry, University of Cape Town, at the 1998 GLOSS Training Course at UCT. Figures by Gillian Spencer.)